

HEAT TRANSFER IN NUCLEATE BOILING (THEORY)

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From a joint solution of the equations describing the individual elementary processes associated with nucleate boiling a theoretical formula is obtained (in criterial form) for the nucleate-boiling heat transfer coefficient of nonmetallic liquids. This formula is compared with the experimental data on water boiling over a broad range of pressures.

Heat transfer in nucleate boiling is a complex process of heat transmission from the heating surface to the vapor-liquid interface, where vapor is produced.

The rate of heat transfer is a function of the physical properties of the heat transfer agent, the saturation pressure (or temperature), and the heat flux. Thus, most authors present the experimental data in the empirical form:

$$\alpha = Aq^n p_s^m,$$

where A is a coefficient depending on the physical properties of the heat transfer agent, and q and p_s are the heat flux and saturation pressure, respectively. Such relations are often given for a very broad range of variation of the parameters q and p_s [1].

This implies the existence of only quantitative changes (with variation of q and p_s), i. e., a change in the number of active sites, bubble breakoff frequency, breakoff diameter, etc., and the absence of any qualitative changes in the boiling pattern. In this case for purposes of a theoretical heat transfer investigation it would be possible to use the same model over the entire range of variation of the parameters.

However, as experiment has shown, in nucleate boiling qualitative as well as quantitative changes are observed. This has been especially well demonstrated by Gaertner [2], who made a photographic study of the process of nucleate pool boiling on a horizontal surface at atmospheric pressure over the entire range of variation of the heat flux from zero to burnout.

Several heat-transfer regions were found to exist depending on the mode of vapor generation:

- 1) Natural convection ($q < 32,200 \text{ W/m}^2$). Total absence of active nucleation sites.
- 2) Nucleate boiling – discrete bubble region ($q = 32,200\text{--}145,000 \text{ W/m}^2$).
- 3) Nucleate boiling – transition region ($q = 145,000\text{--}251,000 \text{ W/m}^2$). Coalescence of discrete bubbles into a vapor structure, a "vapor mushroom" attached to the heating surface by numerous columnar stems of vapor.
- 4) Nucleate boiling – vapor mushroom region ($q = 251,000\text{--}q_b \text{ W/m}^2$).

Gaertner makes the fully justified conclusion that it is impossible to use a single model in analyzing the nucleate boiling process over a broad range of variation of the heat flux.

The transition from discrete bubbles to vapor mushrooms evidently represents a sort of hydrodynamic crisis in vapor removal. When the amount of vapor generated becomes so great that it can no longer be removed from the heating surface in the form of small discrete bubbles, the latter coalesce into mushrooms, and vapor removal is realized in the form of large vapor structures.

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However, vapor removal may be so organized that this crisis does not develop even at high heat loads. In this case vapor generation will correspond to the discrete bubble type. For example, this picture is observed in the case of surface boiling with condensation of the vapor bubbles.

The discrete bubble region also expands when the saturation pressure is increased, as a result of the reduction in the specific volume of the vapor generated.

Thus, nucleate boiling in the discrete bubble region embraces a broad range of the parameters of boiling liquids and is of considerable practical interest in connection with the design of boilers and steam generators.

We have therefore undertaken a theoretical investigation of the boiling process in the discrete bubble region in relation to the rate of heat transfer at the vapor-generating surface.

To simplify the problem, we will consider a single cell on the heating surface of area F , at the center of which an active site is located.

The total amount of heat removed from the cell can be divided into two principal components [3]:

1) Heat removed by convection from the surface wetted by the liquid, i. e., from the entire surface of the cell $F = 1/n$.

2) Heat removed from the part of the surface located within the vapor bubble, i. e., from an area $F'' \approx \pi R_0^2$. This chiefly includes the heat expended on vapor generation at the heating surface and the non-stationary heat transfer associated with the sudden cooling of the wall by the liquid in the process of bubble growth and breakoff.

Heat from the surface (outside the vapor bubble) is transmitted through the boundary layer to the main mass of liquid, from which it is transferred to the rising bubbles and the liquid surface. The principle thermal resistance is offered by the thermal boundary layer at the heating surface. The thermal resistance of the main mass of liquid outside the boundary layer is very small and may be neglected [4].

In this case the heat transfer rate is chiefly determined by the motion of the liquid near the heating surface and its degree of turbulence.

"The displacement of liquid by the growing vapor bubble, the breakoff of the bubbles formed at the wall surface, and their replacement by water from the main bulk of the liquid leads to the very intense perturbation of the boundary layer. As distinct from convective heat transfer without change in the state of aggregation, when the boundary layer directly adjacent to the heating surface is almost stationary liquid, in boiling near active sites the liquid is actively mixed even at the heating surface itself" [5].

Since at any surface washed by a wetting liquid there is at least a thin laminar film, even in the presence of intense turbulent perturbation of the boundary layer, heat removal from the surface will be realized by both turbulent and molecular heat conduction. If we neglect the propagation of heat in a direction parallel to the heating surface, then the heat flux removed from the surface

$$q' = -(\lambda + \lambda_\tau) \frac{dt}{dZ}. \quad (1)$$

Assuming that the mechanism of turbulent transfer in the boundary layer in nucleate boiling is analogous to the mechanism involved in convective heat transfer without change in the state of aggregation, we adopt for solving Eq. (1) the three-layer model of turbulent flow proposed by Karman. The only difference is that we neglect the thermal resistance of the main bulk of the liquid lying outside the boundary layer and set it equal to zero. In this case we take into account only the sum of the thermal resistances of the "viscous" and "intermediate" layers of liquid at the heating surface.

A method of solving an equation analogous to Eq. (1) is given in [6], so we merely present the final result:

$$q' = \frac{\lambda \Delta t v^*}{v \varphi_* (\text{Pr})}, \quad (2)$$

where

$$\varphi_*(Pr) = \frac{1}{\varepsilon\chi'Pr} \ln \frac{1 + \varepsilon Pr(\chi'\eta_2 - 1)}{1 + \varepsilon Pr(\chi'\eta_1 - 1)} + \frac{\eta_1}{4\sqrt{2}b} \left(\ln \frac{1 + b\sqrt{2} + b^2}{1 - b\sqrt{2} + b^2} + 2\operatorname{arctg} \frac{b\sqrt{2}}{1 - b^2} \right).$$

Here, $b = \sqrt{\varepsilon\beta\eta_1 Pr}$.

Taking as the values of the constant coefficients [6] $\varepsilon = 1$, $\beta = 0.032$, $\chi' = 0.2$, $\eta_1 = 6$, $\eta_2 = 30$, we can approximately write

$$\frac{1}{\varphi_*(Pr)} \cong 0.076 Pr^{0.3}. \quad (3)$$

In the range of Pr numbers from 1 to 100 simplified equation (3) gives a result correct to within 8%.

Substituting (3) in (2), we obtain

$$q' = 0.076 \frac{\lambda\Delta tv^*}{\nu} Pr^{0.3}. \quad (4)$$

Here, v^* is the dynamic velocity, which in this case characterizes the intensity of turbulent transfer in the boundary layer.

In the case of convective heat transfer the dynamic velocity is proportional to the flow velocity of the liquid at the boundary of the viscous layer. The proportionality factor is equal to $1/\eta_1$. In boiling, the intensity of the perturbation of the boundary layer is a function not only of the turbulent fluctuations occurring in the main bulk of the liquid as a result of its high velocities, but chiefly depends on the oscillations produced by the growth, separation, and vibration of the bubbles on the heating surface. In this case, too, the dynamic viscosity is clearly proportional to the liquid velocity; however, the proportionality factor may have a value other (higher) than $1/\eta_1$.

In the process of nucleate boiling a bubble growing on the surface displaces the surrounding liquid. This causes near the heating surface a complex motion of the liquid with a velocity and direction that vary with time. However, if we neglect friction of the liquid against the heating surface and consider the motion of the liquid in the cell as quasi-stationary with a constant source at the active site, we can draw a certain analogy between the motion of the liquid during boiling and the motion of an axisymmetric jet in an infinite space [7]. Then the velocity of the liquid at the heating surface

$$W = -4.021 \sqrt{\frac{I_0}{\rho}} \frac{1}{R_1}, \quad (5)$$

$$I_0 = \rho VU; \quad (6)$$

here, V is the volume of liquid displaced by the bubble during its growth,

$$V = \frac{4}{3} \pi \frac{R_0^3}{\tau_0}; \quad (7)$$

U is the mean flow velocity of the displaced liquid normal to the heating surface,

$$U = 2 \left(\frac{dR}{d\tau} \right)_m. \quad (8)$$

Expressions (7) and (8) have been written for a circular bubble with zero contact angle.

The rate of growth of a bubble on the heating surface [3]

$$\frac{dR}{d\tau} = \beta_* \frac{\lambda\Delta t}{r\rho''R}, \quad (9)$$

where β_* is a coefficient equal to 6, whence the mean rate of growth in time τ_0

$$\left(\frac{dR}{d\tau} \right)_m = \frac{R_0}{\tau_0} = 2.6 \frac{\lambda\Delta t}{r\rho''R_0}. \quad (10)$$

Substituting (6) in (5), using (7), (8), and (10), we obtain

$$W = -16 \sqrt{\frac{2}{3}} \pi \cdot 0.21 \cdot 6 \frac{\lambda\Delta t}{r\rho''R_1}. \quad (11)$$

The liquid velocity W is the time-averaged value and does not take the fluctuations into account. The minus sign in Eq. (11) characterizes the direction of motion of the liquid (from the periphery toward the center); since we are interested in the absolute value, and not in the direction of the velocity, it will henceforth be omitted.

Integrating (11) over the surface of the cell from zero to R_n and dividing by the area of the cell $F = \pi R_n^2 = 1/n$, we obtain the mean flow velocity of the liquid at the heating surface

$$W_m = 32 \sqrt{\frac{2}{3}} \pi \cdot 0.21 \cdot 6 \frac{\lambda \Delta t}{r \rho'' R_n} = 9.7 \cdot 6 \frac{\lambda \Delta t}{r \rho''} \sqrt{\pi n}.$$

Denoting the proportionality factor relating the mean velocity of the liquid at the heating surface and the dynamic velocity in nucleate boiling by $1/\eta_b$, we write

$$v^* = \frac{1}{\eta_b} W_m = 9.7 \frac{6}{\eta_b} \frac{\lambda \Delta t}{r \rho''} \sqrt{\pi n}. \quad (12)$$

Substituting v^* from (12) into (4) and multiplying q' by the area of the cell and τ_0 , we find the amount of heat removed from the surface of a single cell by convective heat transfer during the period of bubble growth:

$$Q' = q' \frac{\tau_0}{n} = 0.74 \frac{6}{\eta_b} \frac{\lambda \Delta t}{r \rho'' v} \text{Pr}^{0.3} \sqrt{\frac{\pi}{n}} \lambda \Delta t \tau_0. \quad (13)$$

Upon increase in the saturation pressure, as a result of the increased specific gravity of the vapor, the rate of bubble growth and hence the convective component of heat removal is reduced. At the same time, owing to the increase in the number of active sites the amount of heat going into bubble growth at the heating surface increases.

The amount of heat absorbed by a bubble during its growth on the heating surface (at zero contact angle)

$$Q'' = r \rho'' \frac{4}{3} \pi R_0^3. \quad (14)$$

From (10) there follows

$$r \rho'' R_0^2 = 2 \cdot 6 \lambda \Delta t \tau_0. \quad (15)$$

In [8] the present authors examined the mutual influence of adjacent sites on the breakoff of a bubble from the heating surface. On the basis of a potential flow over the bubble they determined the dynamic pressure of the liquid on the bubble and obtained a formula for the breakoff radius. As follows from this formula, at high pressures the breakoff radius tends to the cell radius. In reality, however, at high pressures the breakoff radius is approximately equal to half the cell radius [9], i. e.,

$$R_0 \approx \frac{1}{2} R_n = \frac{1}{2 \sqrt{\pi n}}. \quad (16)$$

Substituting (15) and (16) in (14), we obtain

$$Q'' = \frac{4}{3} 6 \sqrt{\frac{\pi}{n}} \lambda \Delta t \tau_0. \quad (17)$$

The total amount of heat removed from a single cell is equal to the sum of its components (13) and (17)

$$Q = Q' + Q''. \quad (18)$$

Dividing the total amount of heat by the area of the cell and the total time of bubble nucleation and growth τ_t , we obtain the mean heat flux removed from the heating surface

$$q = Q \frac{n}{\tau_t} = \left(\frac{4}{3} + \frac{0.74}{\eta_b} \frac{\lambda \Delta t}{r \rho'' v} \text{Pr}^{0.3} \right) 6 \sqrt{\pi n} \lambda \Delta t \frac{\tau_0}{\tau_t}. \quad (19)$$

As a rule, the ratio τ_0/τ_t is less than unity and depends on the conditions of the boiling process. The times of nucleation and growth are roughly the same [10], so that τ_0/τ_t may be assumed equal to 0.5.

For the number of active sites we use the formula [11]

$$n = 625 \cdot 10^{-16} L \left(\frac{r \rho'' \Delta t}{T_s \sigma} \right)^3. \quad (20)$$

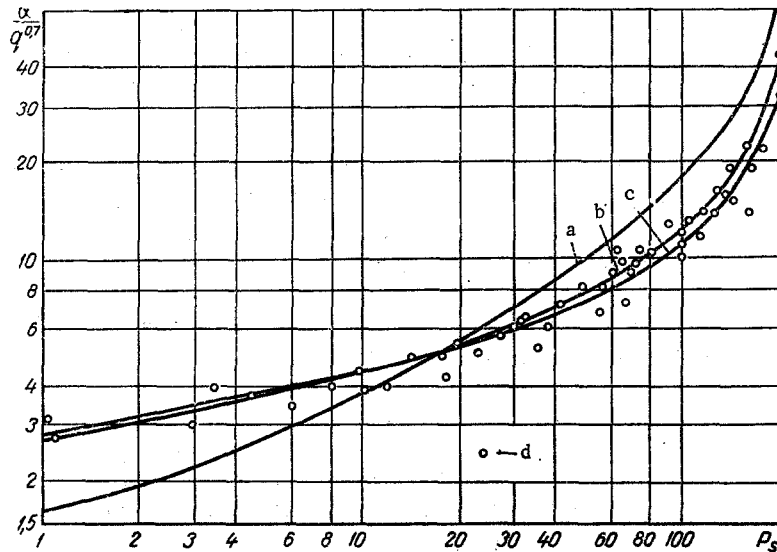


Fig. 1. Comparison of theory with the experimental data for boiling water (α , $\text{m}^2 \cdot \text{deg}$; p_s , bar): a) according to (19), $q = 116,000$ W/m^2 ; b) according to (22), $q = 116,000$ W/m^2 ; c) according to (22), $q = 930,000$ W/m^2 ; d) experimental data of [12].

Together with (20) Eq. (19) gives the relation $q = f(\Delta t)$, which can be used to determine the heat transfer coefficient.

In analyzing the question of the heat transfer rate in boiling we have used the results of theoretical and experimental investigations of the individual elementary processes associated with the nucleate boiling of nonmetallic liquids: bubble nucleation (number of active sites), bubble growth on heating surface (growth rate), breakoff of bubbles from the heating surface (breakoff radius), heat transfer in the boundary layer, etc.

From a joint solution of the equations describing the elementary processes we obtained Eq. (19) which, with (20), can be used to determine the heat transfer accompanying the nucleate boiling of nonmetallic liquids over a broad range of the parameters of the boiling liquids in the discrete bubble region.

Equation (19) was obtained by summing the individual components of heat removal from the heating surface: the heat removed by convection and the heat supplied to the growing bubbles.

As the calculations show, as a rule, in the region of low pressures and small heat fluxes heat removal depends chiefly on convection, while at high pressures the heat is almost completely expended on vapor generation at the heating surface, i. e., on the growth of vapor bubbles.

In Fig. 1 we have plotted the graph of $\alpha/q^{0.7} = f(P)$ on the basis of calculations made in accordance with Eq. (19) at $q = 116,000$ W/m^2 . In the calculations we assumed that $\eta_b = \eta_1 = 6$. A comparison of the results with the experimental data for water [12] (see Fig. 1) reveals a discrepancy between theory and experiment. At low pressures the theoretical heat transfer coefficients are too low, at high pressures too high. The explanation is probably as follows. At low pressures, where convection predominates, only the averaged motion of the liquid near the heating surface, not the pulsations due to bubble growth and breakoff, was taken into account. These pulsations can be approximately allowed for by introducing a higher proportionality factor. Good agreement is obtained at $\eta_b = 1$. In the high-pressure region the values obtained for the amount of heat expended on vapor generation at the heating surface are too high. This is evidently because Eqs. (9) and (20) for the bubble growth rate and the number of active sites give unreal (too high) values in this region. Thus, it is desirable to introduce a reducing factor of 0.3 into the first term of Eq. (19).

With allowance for the above and Eq. (20), Eq. (19) now becomes

$$q = \left(1 + 1.85 \frac{\lambda \Delta t}{r \rho'' v} \text{Pr}^{0.3} \right) 5.32 \cdot 10^{-7} \sqrt{L} \left(\frac{r \rho'' \Delta t}{T_s \sigma} \right)^{1.5} \lambda \Delta t. \quad (21)$$

Equation (21) is in satisfactory agreement with experiment, but it is not convenient for calculating the heat transfer coefficient; we therefore present the following formula, obtained on the basis of a solution of Eq. (21) and more convenient for calculation purposes:

$$Nu_* = 0.003 (Pe_* Kt_*)^{0.6} \sqrt[3]{1 + 260 \frac{\sqrt[3]{Pe_*}}{Kt_* Pr}} \quad (22)$$

The results of calculations based on Eq. (22) at $q = 116,000 \text{ W/m}^2$ and $q = 930,000 \text{ W/m}^2$ are presented in Fig. 1. Good agreement with experiment is observed.

NOTATION

Z	is a coordinate;
I_0	is the momentum flux through jet cross section;
R	is the radius of bubble growing on heating surface;
R_0	is the breakoff radius;
R_n	is the cell radius;
R_1	is the variable radius in cell;
τ	is the time;
τ_0	is the bubble growth time to breakoff point;
τ_t	is the total nucleation and growth time;
ν^*	is the dynamic viscosity of liquid;
Δt	is the wall-liquid temperature difference;
q	is the heat flux to heating wall;
q'	is the heat flux removed from surface by convection;
n	is the number of active sites;
γ''	is the specific gravity of vapor;
γ	is the specific gravity of liquid;
r	is the latent heat of vaporization;
λ	is the thermal conductivity of liquid;
c	is the specific heat of liquid;
σ	is the surface tension;
ν	is the kinematic viscosity of liquid;
ρ	is the density of liquid;
ρ''	is the density of vapor;
η	is the dimensionless distance from wall.

Similarity criteria:

$$Nu_* = \frac{\alpha}{\lambda} \sqrt{\frac{\sigma}{\gamma - \gamma''}}; \quad Pe_* = \frac{qc\rho}{r\rho''\lambda} \sqrt{\frac{\sigma}{\gamma - \gamma''}}; \quad Pr = \frac{\nu c\rho}{\lambda}$$

$$Kt = \frac{(r\rho'')^2}{T_s \sigma c\rho} \sqrt{\frac{\sigma}{\gamma - \gamma''}}; \quad Kt_* = Kt \left(L \sqrt{\frac{\gamma - \gamma''}{\sigma}} \right)^{\frac{1}{3}};$$

L is a coefficient with dimension of length, $L = 1 \text{ m}$.

LITERATURE CITED

1. S. S. Kutateladze and V. M. Borishanskii, Heat Transfer Handbook [in Russian], GÉI (1959).
2. R. F. Gaertner, J. Heat Transfer, Trans. ASME, Series C, No. 1 (1965).
3. D. A. Labuntsov, Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, Énergetika i Transport, No. 1 (1963).
4. G. N. Kruzhilin, Izv. Akad. Nauk SSSR, OTN, No. 5 (1949).
5. M. A. Styrikovich, Boiler Processes [in Russian], GÉI (1954).
6. S. S. Kutateladze, Fundamentals of Heat Transfer Theory [in Russian], Mashgiz (1962).
7. L. G. Loitsyanskii, Mechanics of Liquids and Gases [in Russian], GITTL (1957).
8. V. M. Borishanskii and K. A. Zhakhov, IFZh [Journal of Engineering Physics], No. 4 (1968).
9. D. A. Labuntsov, B. A. Kol'chugin, V. S. Golovin, É. A. Zakharova, and L. N. Vladimirova, in: Heat Transfer in Power Plant Elements [in Russian], Izd. Nauka (1956).
10. M. Jakob, Heat Transfer, Vol. 1, Wiley, New York (1949).

11. K. A. Zhokhov, Trudy TsKTI, No. 91 (1968).
12. V. M. Borishanskii, A. P. Kozyrev, and L. S. Svetlova, Teplofizika Vysokikh Temperatur, 2, No. 1 (1964).